Control of electronic transport in graphene by electromagnetic dressing

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We demonstrated theoretically that the renormalization of the electron energy spectrum near the Dirac point of graphene by a strong high-frequency electromagnetic field (dressing field) drastically depends on polarization of the field. Namely, linear polarization results in an anisotropic gapless energy spectrum, whereas circular polarization leads to an isotropic gapped one. As a consequence, the stationary (dc) electronic transport in graphene strongly depends on parameters of the dressing field: A circularly polarized field monotonically decreases the isotropic conductivity of graphene, whereas a linearly polarized one results in both giant anisotropy of conductivity (which can reach thousands of percents) and the oscillating behavior of the conductivity as a function of the field intensity. Since the predicted phenomena can be observed in a graphene layer irradiated by a monochromatic electromagnetic wave, the elaborated theory opens a substantially new way to control electronic properties of graphene with light.

INTRODUCTION

Since the discovery of graphene [1], it has attracted the persistent interest of the scientific community. Particularly, the influence of an electromagnetic field on the electronic properties of graphene is in the focus of attention [2, 3]. Usually, the electron-field interaction is considered within the regime of weak light-matter coupling, where the electron energy spectrum is assumed to be unperturbed by photons. However, a lot of interesting physical effects can be expected within the regime of strong light-matter coupling, where the electron energy spectrum is strongly modified by a high-frequency electromagnetic field. Following the conventional classification, this regime is jurisdictional to quantum optics which is an established part of modern physics [4, 5]. Therefore, the developing of interdisciplinary research at the border between graphene physics and quantum optics is on the scientific agenda.

The methodology of quantum optics lies at the basis of various exciting fields of modern physics, including quantum information [6], polaritonics [7], quantum teleportation [8, 9], quantum cryptography [10, 11], etc. Particularly, it allows to describe fundamental physical effects (e.g., Bose-Einstein condensation of polaritons [12] and optical bistability [13]) and creates a basis of modern technological applications (e.g., optical logic circuits [14], novel sources of terahertz emission [15], and novel types of lasers [16, 17]). Within the quantum optics approach, the system "electron + strong electromagnetic field" should be considered as a whole. Such a bound electron-field system, which was called "electron dressed by field" (dressed electron), became a commonly used model in modern physics [4, 5]. The field-induced modification of the energy spectrum and wave functions of dressed electrons was discovered many years ago and has been studied in detail in various atomic systems [18–23] and condensed matter [24–33]. In graphene-related research, the attention has been paid to the field-induced modification of energy spectrum of dressed electrons [34–41], optical response of dressed electrons [42], transport of dressed electrons in graphene-based p-n junctions [43] and electronic transport through dressed edge states in graphene [44–48]. As to stationary (dc) transport properties of a spatially homogeneous graphene layer dressed by light, they still await detailed analysis. The present Report is aimed to fill partially this gap at the border between graphene physics and quantum optics.

MODEL

For definiteness, we will restrict our consideration to the case of electron states near the Dirac point of a single graphene sheet subjected to an electromagnetic wave propagating perpendicularly to the graphene plane. Let the graphene sheet lie in the plane (x, y) at $z = 0$, and the wave propagate along the z axis [see Fig. (1)]. Then electronic properties of the graphene are described by the Hamiltonian [2, 3]

$$\hat{\mathcal{H}} = v\boldsymbol{\sigma} \cdot (\hbar\mathbf{k} - e\mathbf{A}), \tag{1}$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$ is the Pauli matrix vector, $\mathbf{k} = (k_x, k_y)$ is the electron wave vector in the graphene plane, v is the electron velocity in graphene near the Dirac point, e is the electron charge, and $\mathbf{A} = (A_x, A_y)$ is the vector potential
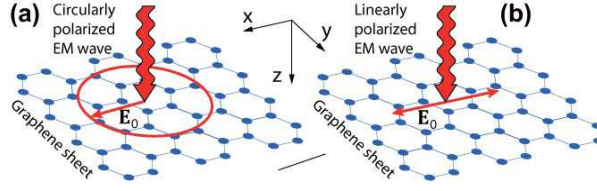
FIG. 1: Sketch of the electron-field system under consideration: The graphene sheet dressed by (a) circularly polarized electromagnetic wave with the amplitude E_0 and (b) linearly polarized one.

of the electromagnetic wave in the graphene plane. In what follows, we will be to assume that the wave frequency, ω, lies far from the resonant frequencies of graphene, $2vk$. Solving the non-stationary Schrödinger equation with the Hamiltonian (1),

$$i\hbar\frac{\partial \psi_{\mathbf{k}}}{\partial t} = \hat{\mathcal{H}}\psi_{\mathbf{k}},$$

we can obtain both the energy spectrum of electrons dressed by the electromagnetic field, $\varepsilon_{\mathbf{k}}$, and their wave functions $\psi_{\mathbf{k}}$ (see technical details of the solving within the Supplementary Information attached to the Report).

For the case of the circularly polarized electromagnetic field with the vector potential

$$\mathbf{A} = \left(\frac{E_0}{\omega}\cos\omega t, \frac{E_0}{\omega}\sin\omega t\right),$$

we arrive at the energy spectrum of the dressed electrons,

$$\varepsilon_{\mathbf{k}} = \pm\sqrt{(\varepsilon_g/2)^2 + (\hbar vk)^2}, \tag{2}$$

where signs "+" and "−" correspond to the conduction band and valence band of graphene, respectively,

$$\varepsilon_g = \sqrt{(\hbar\omega)^2 + (2veE_0/\omega)^2} - \hbar\omega \tag{3}$$

is the field-induced band gap in graphene, E_0 is the amplitude of electric field of the electromagnetic wave, and the field frequency ω is assumed to satisfy the condition of $\omega \gg \sqrt{2veE_0/\hbar}$. Corresponding wave functions of electrons dressed by the circularly-polarized field read as

$$\psi_{\mathbf{k}} = \varphi_{\mathbf{k}}(\mathbf{r})e^{-i\varepsilon_{\mathbf{k}}t/\hbar}\left[\sqrt{\frac{|\varepsilon_{\mathbf{k}}| \mp \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}}|}}e^{-i\theta/2}\,\Phi_1(\mathbf{r}) \pm \sqrt{\frac{|\varepsilon_{\mathbf{k}}| \pm \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}}|}}e^{i\theta/2}\Phi_2(\mathbf{r})\right], \tag{4}$$

where $\mathbf{r} = (x, y)$ is the electron radius-vector in the graphene plane, $\Phi_{1,2}(\mathbf{r})$ are the known basic functions of the graphene Hamiltonian (the periodical functions arisen from atomic π-orbitals of the two crystal sublattices of graphene) [2], $\varphi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}/\sqrt{S}$ is the plane electron wave, S is the graphene area, and θ is the azimuth angle of electron in the space of wave vector, $\mathbf{k} = (k\cos\theta, k\sin\theta)$.

In the case of linearly polarized electromagnetic field with the vector potential

$$\mathbf{A} = \left(\frac{E_0}{\omega}\cos\omega t, 0\right)$$

directed along the x axis, the energy spectrum of the dressed electrons reads as

$$\varepsilon_{\mathbf{k}} = \pm\hbar vkf(\theta), \tag{5}$$

and the corresponding wave functions of dressed electrons are

$$\psi_{\mathbf{k}} = \left([\Phi_1(\mathbf{r}) \pm \Phi_2(\mathbf{r})] \times e^{\pm i(veE_0/\hbar\omega^2)\sin\omega t} - i\frac{\sin\theta}{\cos\theta + f(\theta)}J_0\left(\frac{2veE_0}{\hbar\omega^2}\right) \times [\Phi_1(\mathbf{r}) \mp \Phi_2(\mathbf{r})]\,e^{\mp i(veE_0/\hbar\omega^2)\sin\omega t}\right)$$

$$\times\ \varphi_{\mathbf{k}}(\mathbf{r})e^{-i\varepsilon_{\mathbf{k}}t/\hbar}\sqrt{\frac{\cos\theta + f(\theta)}{4f(\theta)}}, \tag{6}$$

where

$$f(\theta) = \sqrt{\cos^2 \theta + J_0^2 \left(\frac{2veE_0}{\hbar\omega^2}\right) \sin^2 \theta}, \tag{7}$$

$J_0(z)$ is the Bessel function of the first kind, and the field frequency ω is assumed to satisfy the condition of $\omega \gg vk$.
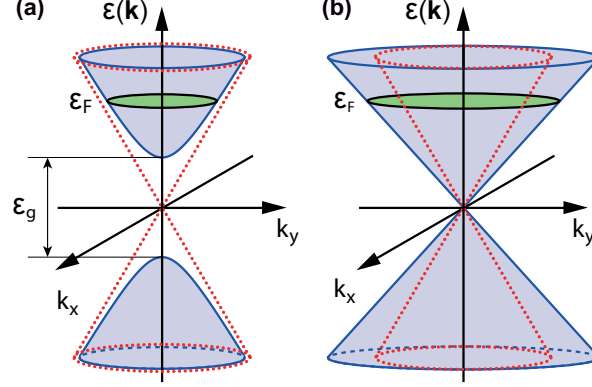

FIG. 2: The energy spectrum of dressed electrons in graphene for the dressing field with different polarizations: (a) circularly polarized dressing field; (b) dressing field polarized along the x axis. The energy spectrum of electrons in absence of the dressing field is plotted by the dotted lines and ε_F is the Fermi energy.

The energy spectra of dressed electrons, (2) and (5), are pictured schematically in Fig. 2. As to a consistent derivation of Eqs. (2)–(7), it can be found within the Supplementary Information attached to the Report. In order to verify the derived expressions, it should be stressed that the energy spectrum of electrons dressed by a classical circularly polarized field, which is given by Eq. (2), exactly coincides with the energy spectrum of electrons dressed by a quantized field in the limit of large photon occupation numbers [36]. This can serve as a proof of physical correctness of the presented approach elaborated for a classical dressing field.

In order to calculate transport properties of dressed electrons, we have to solve the scattering problem for nonstationary electron states (4) and (6). Following the scattering theory for dressed conduction electrons [32], the problem comes to substituting the wave functions of dressed electrons (4) and (6) into the conventional expression for the Born scattering probability [49]. Assuming a total scattering potential in a graphene sheet, $U(\mathbf{r})$, to be smooth within an elementary crystal cell of graphene, we can write its matrix elements as

$$\langle \Phi_i(\mathbf{r})\varphi_{\mathbf{k}'}(\mathbf{r}) | U(\mathbf{r}) | \Phi_j(\mathbf{r})\varphi_{\mathbf{k}}(\mathbf{r}) \rangle \approx U_{\mathbf{k}'\mathbf{k}}\delta_{ij},$$

where $U_{\mathbf{k}'\mathbf{k}} = \langle \varphi_{\mathbf{k}'}(\mathbf{r}) | U(\mathbf{r}) | \varphi_{\mathbf{k}}(\mathbf{r}) \rangle$, and δ_{ij} is the Kronecker delta. As a result, the Born scattering probability for dressed electronic states in graphene takes the form

$$w_{\mathbf{k}'\mathbf{k}} = \frac{2\pi}{\hbar} |\chi_{\mathbf{k}'\mathbf{k}}|^2 |U_{\mathbf{k}'\mathbf{k}}|^2 \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}), \tag{8}$$

where

$$\chi_{\mathbf{k}'\mathbf{k}} = \sqrt{\frac{|\varepsilon_{\mathbf{k}'}| - \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}'}|}} \sqrt{\frac{|\varepsilon_{\mathbf{k}}| - \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}}|}} e^{i(\theta'-\theta)/2} + \sqrt{\frac{|\varepsilon_{\mathbf{k}'}| + \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}'}|}} \sqrt{\frac{|\varepsilon_{\mathbf{k}}| + \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}}|}} e^{-i(\theta'-\theta)/2} \tag{9}$$

for the case of circularly polarized dressing field, and

$$\chi_{\mathbf{k}'\mathbf{k}} = \sqrt{\frac{\cos(\theta') + f(\theta')}{2f(\theta')}} \sqrt{\frac{\cos(\theta) + f(\theta)}{2f(\theta)}} \left[1 + \frac{\sin(\theta')}{\cos(\theta') + f(\theta')} \frac{\sin(\theta)}{\cos(\theta) + f(\theta)} J_0^2 \left(\frac{2veE_0}{\hbar\omega^2}\right)\right] \tag{10}$$

for the case of linearly polarized dressing field.

In what follows, we will assume that the wave frequency, ω, meets the condition

$$\omega\tau_0 \gg 1, \tag{11}$$

where τ_0 is the electron relaxation time in an unirradiated graphene, which should be considered as a phenomenological parameter taken from experiments. It is well-known that the intraband (collisional) absorption of wave energy by conduction electrons is negligibly small under condition (11) (see, e.g., Refs. [32, 50, 51]). Thus, the considered electromagnetic wave can be treated as a purely dressing field which can be neither absorbed nor emitted by conduction electrons. As a consequence, the field does not heat the electron gas and, correspondingly, the electrons are in thermodynamic equilibrium with a thermostat. Therefore, electron distribution under the condition (11) can be described by the conventional Fermi-Dirac function, where the energy of "bare" electron should be replaced with the energy of dressed electron (2),(5). Substituting both this Fermi-Dirac function and the scattering probability (8) into the conventional kinetic Boltzmann equation, we can analyze the stationary (dc) transport properties of dressed electrons in graphene. Within this approach, we take into account the two key physical factors arisen from a dressing field: (i) modification of the electron energy spectra (2) and (5) by the dressing field; (ii) renormalization of the electron scattering probability (8)–(10) by the dressing field.

RESULTS AND DISCUSSION

Let us focus our attention on the dc conductivity of the dressed electrons. Generally, the density of the conduction electrons can be tuned by applying a bias voltage which fixes the Fermi energy, ε_F, of electron gas [2]. Assuming the Fermi energy to be in the conduction band and the temperature to be zero, let us apply a stationary (dc) electric field $\mathbf{E} = (E_x, E_y)$ to the graphene sheet. It follows from the conventional Boltzmann equation for conduction electrons (see, e.g., Refs. [2, 52]) that the electric current density, $\mathbf{J}$, is given by the expression

$$\mathbf{J} = \frac{e^2}{\pi^2} \int_{\mathbf{k}} [\mathbf{E} \cdot \mathbf{v}(\mathbf{k})] \, \tau(\mathbf{k}) \mathbf{v}(\mathbf{k}) \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) d^2\mathbf{k}, \tag{12}$$

where $\mathbf{v}(\mathbf{k}) = (1/\hbar)\nabla_{\mathbf{k}}\varepsilon_{\mathbf{k}}$ is the electron velocity, and $\tau(\mathbf{k})$ is the relaxation time. In the most general case of anisotropic electron scattering, this relaxation time is given by the equation [53]

$$\frac{1}{\tau(\mathbf{k})} = \sum_{\mathbf{k}'} \left[1 - \frac{\tau(\mathbf{k}')\mathbf{E} \cdot \mathbf{v}(\mathbf{k}')}{\tau(\mathbf{k})\mathbf{E} \cdot \mathbf{v}(\mathbf{k})} \right] w_{\mathbf{k}'\mathbf{k}}. \tag{13}$$

Substituting the scattering probability of dressed electron (8) into Eq. (13), we can obtain from Eqs. (12)–(13) the conductivity of dressed graphene, $\sigma_{ij} = J_i/E_j$.

To simplify calculations, let us consider the electron scattering within the s-wave approximation [49], where the matrix elements $U_{\mathbf{k}'\mathbf{k}}$ do not depend on the angle $\theta_{\mathbf{k}'\mathbf{k}} = (\widehat{\mathbf{k}', \mathbf{k}})$. Substituting the probability (8) into Eqs. (12)–(13), we arrive at the isotropic conductivity of a graphene dressed by a circularly polarized field, $\sigma_c = \sigma_{xx} = \sigma_{yy}$, which is given by the expression

$$\frac{\sigma_c}{\sigma_0} = \frac{1 - \varepsilon_g^2/4\varepsilon_F^2}{1 + 3\varepsilon_g^2/4\varepsilon_F^2}, \tag{14}$$

where $\varepsilon_F \geq \varepsilon_g$. It is seen in Fig. 3a that the conductivity (14) monotonically decreases with increasing field intensity $I_0 = \epsilon_0 E_0^2 c/2$. Physically, this behavior is a consequence of decreasing Fermi velocity, $\mathbf{v}_F(\mathbf{k}) = (1/\hbar)\nabla_{\mathbf{k}}\varepsilon_{\mathbf{k}}|_{\varepsilon=\varepsilon_F}$, with increasing field amplitude E_0 (see Fig. 2a). For the case of a dressing field linearly polarized along the x axis, the conductivity is plotted in Figs. 3b–3c. There are the two main features of the conductivity as a function of the dressing field intensity: Firstly, the conductivity oscillates, and, secondly, the giant anisotropy of the conductivity, $\sigma_{xx}/\sigma_{yy} \sim 10$ appears (see the insert in Fig. 3b). The oscillating behavior arises from the Bessel functions which take place in both the energy spectrum (5) and the scattering probability (8). As to the conductivity anisotropy, it is caused by the field-induced anisotropy of the energy spectrum (5). Namely, the linearly polarized dressing field turns the round (isotropic) Fermi line of unperturbed graphene into the strongly anisotropic ellipse line (see Fig. 2b). As a result, the Fermi velocities of dressed electrons along the x, y axes are strongly different and the discussed anisotropic conductivity appears. It should be stressed that the aforementioned features of electronic properties are typical exclusively for linear electron dispersion and, correspondingly, do not take place in a dressed electron gas with parabolic dispersion [33]. To avoid misunderstandings, it should be noted also that the zeros of conductivity in Fig. 3b lie within physically irrelevant areas pictured by dashed lines. Formally, these irrelevant areas correspond to the broken condition $\omega \gg vk_F$, which is crucial for the correctness of the energy spectrum (5) at the Fermi energy. Thus, the zeros have no physical meaning and should be ignored.
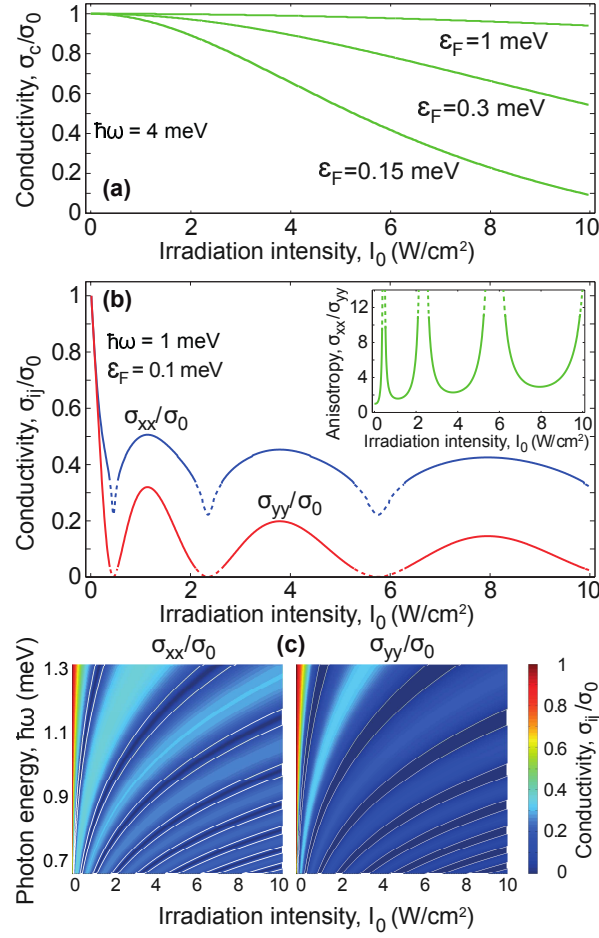
FIG. 3: The conductivity of dressed electrons in graphene for the dressing field with the different polarizations: (a) circularly polarized dressing field; (b)–(c) dressing field polarized along the x axis. Physically relevant regions of the field parameters, where the developed theory is applicable, correspond to the solid lines in the plot (b) and wide areas between the dashed lines in the plot (c).

It is seen in Fig. 3 that the behavior of conductivity is qualitatively different for the dressing field with different polarizations. Physically, the strong polarization dependence of electronic transport follows directly from the strong polarization dependence of energy spectrum of dressed electrons. Namely, the energy spectrum of electrons dressed by a circularly polarized field (2) is isotropic and has the field-induced gap (3) at the Dirac point. In contrast, the energy spectrum of electrons dressed by the linearly polarized field (5) is gapless and has the field-induced anisotropy arisen from the Bessel function in Eq. (7). These differences in the spectra (2) and (5) lead to the discussed difference of transport for electrons dressed by circularly polarized light and linearly polarized one. It should be noted that an electromagnetic field can open energy gaps within conduction and valence bands at electron wave vectors $\mathbf{k} \neq 0$ (see, e.g., Refs. [38–41]). These gaps arise from the optical (ac) Stark effect and take place at resonant points of the Brillouin zone, where the condition of $\omega = 2vk$ is satisfied. Certainly, the basic expressions (2)–(7) are not applicable near the Stark gaps. However, these gaps lie far from the Dirac point in the case of high-frequency dressing field. Therefore, they do not influence on low-energy electronic transport under consideration.

CONCLUSIONS

We have shown that the transport properties of electrons in graphene are strongly affected by a dressing field. Namely, a circularly polarized dressing field monotonically decreases the isotropic conductivity of graphene, whereas a linearly polarized dressing field results in the oscillating behavior of the conductivity and its giant anisotropy. As a result, the dc transport properties of graphene can be effectively controlled by a strong high-frequency electromagnetic

field. From the viewpoint of possible applications, the discussed effect can make graphene more tunable. Particularly, the switching times for conductivity of graphene controlled by a high-frequency field are expected to be shorter then for the case of conventional electrostatic control of conductivity by gate electrodes. This can create physical prerequisites for novel graphene-based optoelectronic devices.

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AUTHOR CONTRIBUTIONS

O.V.K. and I.A.S. formulated the physical problem under consideration and derived analytical solutions of the problem. K.K. and S.M. analyzed the basic expressions describing the problem, performed numerical calculations and plotted figures. O.V.K. and K.K. wrote the paper. All co-authors taken part in discussions of used physical models and obtained results.

ADDITIONAL INFORMATION

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SUPPLEMENTARY INFORMATION:
Full and consistent derivation of the energy spectrum and wave functions of dressed electrons in graphene

Let us consider a graphene sheet which lies in the plane (x, y) at $z = 0$ and is subjected to an electromagnetic wave propagating along the z axis (dressing electromagnetic field). Then electronic properties of the graphene are described by the Hamiltonian

$$\hat{\mathcal{H}} = v\boldsymbol{\sigma} \cdot (\hbar\mathbf{k} - e\mathbf{A}), \tag{15}$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$ is the Pauli matrix vector, $\mathbf{k} = (k_x, k_y)$ is the electron wave vector in the graphene plane, v is the electron velocity in graphene near the Dirac point, e is the electron charge, and $\mathbf{A} = (A_x, A_y)$ is the vector potential of the electromagnetic wave in the graphene plane. Solving the Schrödinger equation with the Hamiltonian (15), we can find the energy spectrum of dressed electrons and their wave functions as follows.

I. Circularly polarized dressing field

For the case of circularly polarized electromagnetic wave, its vector potential $\mathbf{A} = (A_x, A_y)$ can be written as

$$\mathbf{A} = \left(\frac{E_0}{\omega} \cos\omega t, \frac{E_0}{\omega} \sin\omega t \right), \tag{16}$$

where E_0 is the electric field amplitude of the wave, and ω is the wave frequency. Then the Hamiltonian (15) is

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_k, \tag{17}$$

where

$$\hat{\mathcal{H}}_0 = \left(-\frac{veE_0}{\omega} \right) \begin{pmatrix} 0 & e^{-i\omega t} \\ e^{i\omega t} & 0 \end{pmatrix}, \tag{18}$$

and

$$\hat{\mathcal{H}}_k = \begin{pmatrix} 0 & v(\hbar k_x - i\hbar k_y) \\ v(\hbar k_x + i\hbar k_y) & 0 \end{pmatrix}. \tag{19}$$

The nonstationary Schrödinger equation with the Hamiltonian (18),

$$i\hbar \frac{\partial \psi_0}{\partial t} = \hat{\mathcal{H}}_0 \psi_0, \tag{20}$$

describes the time evolution of electron states in the Dirac point ($\mathbf{k} = 0$). The exact solutions of the equation (20) can be sought in the form

$$\psi_0 = e^{-i\alpha t/\hbar}[A\Phi_1'(\mathbf{r})e^{-i\omega t/2} + B\Phi_2'(\mathbf{r})e^{i\omega t/2}], \tag{21}$$

where $\Phi_{1,2}'(\mathbf{r})$ are the basic functions of the 2×2 matrix Hamiltonian (15), and α, A and B are the sought parameters. Substituting the wave function (21) into the Schrödinger equation (20), we arrive at the system of algebraic equations

$$A\left(\alpha + \frac{\hbar\omega}{2} \right) + B\left(\frac{veE_0}{\omega} \right) = 0$$
$$A\left(\frac{veE_0}{\omega} \right) + B\left(\alpha - \frac{\hbar\omega}{2} \right) = 0. \tag{22}$$

The condition of nontrivial solution of the system (22),

$$\begin{vmatrix} \alpha + \frac{\hbar\omega}{2} & \frac{veE_0}{\omega} \\ \frac{veE_0}{\omega} & \alpha - \frac{\hbar\omega}{2} \end{vmatrix} = 0,$$

gives the two different parameters, $\alpha = \pm\Omega/2$, where

$$\Omega = \sqrt{(\hbar\omega)^2 + \left(\frac{2veE_0}{\omega}\right)^2}. \tag{23}$$

Therefore, there are two sets of solutions of the system (41), which correspond to these two parameters and satisfy the normalization condition, $|A|^2 + |B|^2 = 1$. As a result, there are two wave functions (21),

$$\psi_0^\pm = e^{\pm i\Omega t/2\hbar}\left[\sqrt{\frac{\Omega \pm \hbar\omega}{2\Omega}}\Phi_1'(\mathbf{r})e^{-i\omega t/2} \pm \frac{e}{|e|}\sqrt{\frac{\Omega \mp \hbar\omega}{2\Omega}}\Phi_2'(\mathbf{r})e^{i\omega t/2}\right], \tag{24}$$

which exactly describe electron states of irradiated graphene in the Dirac point ($\mathbf{k} = 0$). Since the two wave functions (24) are the complete function system for any time t, we can seek the solution of the Schrödinger equation with the full Hamiltonian (17) as an expansion

$$\psi_{\mathbf{k}} = a^+(t)\psi_0^+ + a^-(t)\psi_0^-. \tag{25}$$

Substituting the expansion (25) into the Schrödinger equation with the full Hamiltonian (17),

$$i\hbar\frac{\partial\psi_{\mathbf{k}}}{\partial t} = \hat{\mathcal{H}}\psi_{\mathbf{k}}, \tag{26}$$

we arrive at the system of two differential equations for the coefficients $a^\pm(t)$,

$$
\begin{aligned}
i\dot{a}^+(t) &= v\frac{e}{|e|}\left[\frac{W_0}{\Omega}(k_x\cos\omega t + k_y\sin\omega t)a^+(t) - (k_x - ik_y)\left(\frac{\Omega + \hbar\omega}{2\Omega}\right)e^{-i(\Omega/\hbar - \omega)t}a^-(t)\right. \\
&\quad \left. + (k_x + ik_y)\left(\frac{\Omega - \hbar\omega}{2\Omega}\right)e^{-i(\Omega/\hbar + \omega)t}a^-(t)\right], \\
i\dot{a}^-(t) &= -v\frac{e}{|e|}\left[\frac{W_0}{\Omega}(k_x\cos\omega t + k_y\sin\omega t)a^-(t) + (k_x + ik_y)\left(\frac{\Omega + \hbar\omega}{2\Omega}\right)e^{i(\Omega/\hbar - \omega)t}a^+(t)\right. \\
&\quad \left. - (k_x - ik_y)\left(\frac{\Omega - \hbar\omega}{2\Omega}\right)e^{i(\Omega/\hbar + \omega)t}a^+(t)\right],
\end{aligned}
\tag{27}
$$

where

$$W_0 = \frac{2veE_0}{\omega}$$

is the characteristic kinetic energy of rotational electron motion induced by the circularly polarized field. In what follows, we will assume that the field frequency ω is high enough to satisfy the condition

$$W_0/\hbar\omega \ll 1. \tag{28}$$

Then we have

$$\frac{W_0}{\Omega} \approx \frac{W_0}{\hbar\omega} \approx 0, \quad \frac{\Omega + \hbar\omega}{2\Omega} \approx 1, \quad \frac{\Omega - \hbar\omega}{2\Omega} \approx 0, \tag{29}$$

and, correspondingly, Eqs. (27) take the form

$$
\begin{aligned}
i\dot{a}^+(t) &= -v\frac{e}{|e|}(k_x - ik_y)e^{-i(\Omega - \hbar\omega)t/\hbar}a^-(t), \\
i\dot{a}^-(t) &= -v\frac{e}{|e|}(k_x + ik_y)e^{i(\Omega - \hbar\omega)t/\hbar}a^+(t).
\end{aligned}
\tag{30}
$$

Let us seek solutions of Eqs. (30) as $a_c^\pm(t) = C^\pm e^{-i(\pm\Omega/2\mp\hbar\omega/2+\varepsilon_{\mathbf{k}})t/\hbar}$, where $C^\pm$ and $\varepsilon_{\mathbf{k}}$ are the sought parameters. Solving the system of equations (30), we arrive at

$$\varepsilon_{\mathbf{k}} = \pm\sqrt{(\varepsilon_g/2)^2 + (\hbar vk)^2}, \tag{31}$$

and

$$\varepsilon_g = \sqrt{(\hbar\omega)^2 + \left(\frac{2veE_0}{\omega}\right)^2} - \hbar\omega. \tag{32}$$

Following the conventional terminology, Eq. (31) describes the spectrum of quasienergies of electrons in graphene dressed by a circularly polarized field, where signs "+" and "−" correspond to conduction band and valence band, respectively. The two wave functions (25), which correspond to the two energy branches (31), can be written as

$$\psi_{\mathbf{k}} = e^{-i\varepsilon_{\mathbf{k}}t/\hbar} \left[\pm\sqrt{\frac{|\varepsilon_{\mathbf{k}}| \mp \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}}|}} e^{-i\theta/2} \left(\frac{e}{|e|}\sqrt{\frac{\Omega + \hbar\omega}{2\Omega}} \Phi_1'(\mathbf{r}) + \sqrt{\frac{\Omega - \hbar\omega}{2\Omega}} \Phi_2'(\mathbf{r})e^{i\omega t} \right) \right.$$
$$\left. - \sqrt{\frac{|\varepsilon_{\mathbf{k}}| \pm \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}}|}} e^{i\theta/2} \left(\sqrt{\frac{\Omega - \hbar\omega}{2\Omega}} \Phi_1'(\mathbf{r})e^{-i\omega t} - \frac{e}{|e|}\sqrt{\frac{\Omega + \hbar\omega}{2\Omega}} \Phi_2'(\mathbf{r}) \right) \right], \tag{33}$$

where θ is the azimuth angle of the wave vector, $\mathbf{k} = (k\cos\theta, k\sin\theta)$. Let us write the basis functions $\Phi_{1,2}'(\mathbf{r})$ in the conventional Bloch form, $\Phi_{1,2}'(\mathbf{r}) = \Phi_{1,2}(\mathbf{r})\varphi_{\mathbf{k}}(\mathbf{r})$, where $\Phi_{1,2}(\mathbf{r})$ are the periodical functions arisen from atomic π-orbitals of the two crystal sublattices of graphene, $\varphi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}/\sqrt{S}$ is the plane electron wave, and S is the graphene area. Keeping in mind inequalities (29), we arrive from (33) to the sought wave functions of dressed electrons in the final form,

$$\psi_{\mathbf{k}} = \varphi_{\mathbf{k}}(\mathbf{r})e^{-i\varepsilon_{\mathbf{k}}t/\hbar} \left[\sqrt{\frac{|\varepsilon_{\mathbf{k}}| \mp \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}}|}} e^{-i\theta/2} \, \Phi_1(\mathbf{r}) \pm \sqrt{\frac{|\varepsilon_{\mathbf{k}}| \pm \varepsilon_g/2}{2|\varepsilon_{\mathbf{k}}|}} e^{i\theta/2} \Phi_2(\mathbf{r}) \right]. \tag{34}$$

II. Linearly polarized dressing field

For the case of electromagnetic wave linearly polarized along the x axis, its vector potential $\mathbf{A} = (A_x, A_y)$ can be written as

$$\mathbf{A} = \left(\frac{E_0}{\omega}\cos\omega t, 0 \right). \tag{35}$$

Then the Hamiltonian (15) is

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_k, \tag{36}$$

where

$$\hat{\mathcal{H}}_0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \frac{veE_0}{\omega} \cos\omega t. \tag{37}$$

and

$$\hat{\mathcal{H}}_k = \begin{pmatrix} 0 & v(\hbar k_x - i\hbar k_y) \\ v(\hbar k_x + i\hbar k_y) & 0 \end{pmatrix}. \tag{38}$$

The nonstationary Schrödinger equation with the Hamiltonian (37),

$$i\hbar\frac{\partial\psi_0}{\partial t} = \hat{\mathcal{H}}_0\psi_0, \tag{39}$$

describes the time evolution of electron states in the Dirac point ($\mathbf{k} = 0$). The exact solutions of this Schrödinger equation can be sought in the form

$$\psi_0 = [A\Phi_1'(\mathbf{r}) + B\Phi_2'(\mathbf{r})] \exp\left[-i\alpha\frac{veE_0}{\hbar\omega^2}\sin\omega t \right], \tag{40}$$

where α, A and B are the sought parameters. Substituting the wave function (40) into the Schrödinger equation (39) with the Hamiltonian (37), we arrive at the system of algebraic equations

$$
\begin{aligned}
A\alpha + B &= 0 \\
A + B\alpha &= 0.
\end{aligned}
$$
(41)

The condition of nontrivial solution of the system (41),

$$
\begin{vmatrix} \alpha & 1 \\ 1 & \alpha \end{vmatrix} = 0,
$$

gives the two different parameters, $\alpha = \pm 1$. Therefore, there are two sets of solutions of the system (41), which correspond to these two parameters and satisfy the normalization condition, $|A|^2 + |B|^2 = 1$:

$$
A = B = \frac{1}{\sqrt{2}},
$$
$$
A = -B = \frac{1}{\sqrt{2}}.
$$
(42)

As a result, there are two wave functions (40),

$$
\psi_0^\pm = \frac{1}{\sqrt{2}} \left[\Phi_1'(\mathbf{r}) \pm \Phi_2'(\mathbf{r}) \right] \exp\left[\pm i \frac{veE_0}{\hbar\omega^2} \sin\omega t \right],
$$
(43)

which exactly describe electron states of irradiated graphene in the Dirac point ($\mathbf{k} = 0$). Since the two wave functions (43) are the complete function system for any time t, we can seek the solution of the nonstationary Schrödinger equation with the full Hamiltonian (36) as an expansion

$$
\psi_{\mathbf{k}} = a^+(t)\psi_0^+ + a^-(t)\psi_0^-.
$$
(44)

Substituting the expansion (44) into this Schrödinger equation with the full Hamiltonian (36),

$$
i\hbar \frac{\partial \psi_{\mathbf{k}}}{\partial t} = \hat{\mathcal{H}}\psi_{\mathbf{k}},
$$
(45)

we arrive at the system of two differential equations for the coefficients $a^\pm(t)$,

$$
\begin{aligned}
i\dot{a}^+(t) &= vk_x a^+(t) + ivk_y a^-(t) \exp\left[-i\frac{2veE_0}{\hbar\omega^2} \sin\omega t \right], \\
i\dot{a}^-(t) &= -ivk_y a^+(t) \exp\left[i\frac{2veE_0}{\hbar\omega^2} \sin\omega t \right] - vk_x a^-(t).
\end{aligned}
$$
(46)

It follows from the Floquet's theorem that the functions $a^\pm(t)$ can be written as

$$
a^\pm(t) = e^{-i\varepsilon_{\mathbf{k}} t/\hbar} \widetilde{a}^\pm(t),
$$
(47)

where $\varepsilon_{\mathbf{k}}$ is the electron quasienergy in the irradiated graphene (the energy spectrum of dressed electrons), and $\widetilde{a}^\pm(t)$ are the periodical functions with the period $T = 2\pi/\omega$. Let us apply the Jacobi-Anger expansion,

$$
e^{iz\sin\gamma} = \sum_{n=-\infty}^{\infty} J_n(z) e^{in\gamma},
$$

to the exponents in the right side of Eqs. (46) and expand the periodical functions $\widetilde{a}^\pm(t)$ into the Fourier series

$$
\widetilde{a}^\pm(t) = \sum_{n=-\infty}^{\infty} c_n^\pm e^{in\omega t}.
$$

Then the differential equations (46) can be transformed to the algebraic equations

$$\left(vk_x - \frac{\varepsilon_{\mathbf{k}}}{\hbar} - n\omega\right)c_n^+ + ivk_y \sum_{m=-\infty}^{\infty} c_{n-m}^- J_m\left(-\frac{2veE_0}{\hbar\omega^2}\right) = 0,$$

$$\left(vk_x + \frac{\varepsilon_{\mathbf{k}}}{\hbar} + n\omega\right)c_n^- + ivk_y \sum_{m=-\infty}^{\infty} c_{n-m}^+ J_m\left(\frac{2veE_0}{\hbar\omega^2}\right) = 0, \qquad (48)$$

where $J_m(z)$ is the Bessel function of the first kind. Equations (48) can be easily solved in the case of high-frequency field satisfying the conditions

$$\hbar\omega \gg \varepsilon_{\mathbf{k}}, \ \omega \gg vk. \qquad (49)$$

In this simplest case, Eqs. (48) for $n \neq 0$ can be reduced to the equation

$$c_n^\pm \approx i\frac{vk_y}{n\omega} \sum_{m=-\infty}^{\infty} c_{n-m}^\mp J_m\left(\mp\frac{2veE_0}{\hbar\omega^2}\right). \qquad (50)$$

Keeping in mind that $|c_n^\pm| \leq 1$ and $|J_n(z)| \leq 1$, Eq. (50) leads to the evident solution $c_{n\neq0}^\pm \approx 0$. After substitution of this solution into Eqs. (48), the expressions (48) are reduced to the two equations

$$\left(vk_x - \frac{\varepsilon_{\mathbf{k}}}{\hbar}\right)c_0^+ + ivk_y c_0^- J_0\left(\frac{2veE_0}{\hbar\omega^2}\right) = 0,$$

$$\left(vk_x + \frac{\varepsilon_{\mathbf{k}}}{\hbar}\right)c_0^- + ivk_y c_0^+ J_0\left(\frac{2veE_0}{\hbar\omega^2}\right) = 0. \qquad (51)$$

Solving the system of the two algebraic equations (51) accurately, we can easily obtain both the coefficients $c_0^\pm$ and the energy spectrum of dressed electrons,

$$\varepsilon_{\mathbf{k}} = \pm\hbar vk f(\theta), \qquad (52)$$

where

$$f(\theta) = \sqrt{\cos^2\theta + J_0^2\left(\frac{2veE_0}{\hbar\omega^2}\right)\sin^2\theta}.$$

Correspondingly, the sought wave functions of dressed electrons (44) take the final form

$$\psi_{\mathbf{k}} = \varphi_{\mathbf{k}}(\mathbf{r})e^{-i\varepsilon_{\mathbf{k}}t/\hbar}\sqrt{\frac{\cos\theta + f(\theta)}{4f(\theta)}}\left([\Phi_1(\mathbf{r}) \pm \Phi_2(\mathbf{r})]\,e^{\pm i(veE_0/\hbar\omega^2)\sin\omega t}\right.$$

$$\left.- i\frac{\sin\theta}{\cos\theta + f(\theta)}J_0\left(\frac{2veE_0}{\hbar\omega^2}\right)[\Phi_1(\mathbf{r}) \mp \Phi_2(\mathbf{r})]\,e^{\mp i(veE_0/\hbar\omega^2)\sin\omega t}\right). \qquad (53)$$

Taking into account Eq. (52), the two conditions (49) can be reduced to the solely condition,

$$\omega \gg vk, \qquad (54)$$

which describes borders of applicability of Eqs. (52) and (53).